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TECHNICAL NOTE

EFFICIENCY OF STRAIGHT FINS COOLED BY NATURAL OR FORCED CONVECTION

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NOMENCLATURE

c, constant for natural convection, equation (3);
C, constant for forced convection, equation (2);
g, acceleration of gravity;
$\bar{h}$, average heat transfer coefficient;
k, thermal conductivity of fin;
k_f, thermal conductivity of fluid;
l, transverse length of fin;
m, n, constants, equation (2);
Pr, Prandtl number;
q, convective heat flux;
p, fin thickness parameter, equation (11);
Re, Reynolds number, $U_\infty x/\nu$;
s, dummy variable in vertical direction;
t, fin half thickness;
t_0, fin half thickness at the base;
T, fin temperature;
T_w, fin base temperature;
T_∞, free-stream temperature;
U_∞, free-stream velocity;
x, vertical coordinate;
x_*, dimensionless coordinate for natural convection, equation (16);
X_*, dimensionless coordinate for forced flow, equation (8);
y, transverse coordinate;
y_*, dimensionless coordinate, y/l.

Greek symbols

α, γ, constants, equation (2);
β, volumetric expansion coefficient;
η, fin efficiency, ϕ/ϕ_i;
ν, kinematic viscosity;
θ, dimensionless temperature, $(T-T_\infty)/(T_w-T_\infty)$;
ϕ, total heat flux of fin;
ϕ_i, total heat flux of an ideal isothermal fin;
ΔT, temperature difference, $T - T_\infty$;
ΔT_w, base temperature difference, $T_w - T_\infty$.

1. INTRODUCTION

EXTENDED surfaces are used in applications to improve heat transfer. When analysing fins it is usually assumed that heat transfer coefficients are specified in advance. Assuming this, the temperature distribution of a fin with a known geometry is easily obtained analytically or numerically. Efficiency charts of different fins obtained in this way are found in the literature [1].

In actual practice the heat transfer coefficient is not specified in advance. Especially when natural convection is considered, the heat transfer coefficient is greatly affected by the surface temperature distribution of a fin. Thus there exists a highly coupled interaction between convection in a fluid and conduction in a fin. Heat transfer of a fin is only obtained accurately by solving simultaneously convection and conduction. Although conjugated heat transfer problems have been much dealt with in the literature, only a few articles on fins have taken the coupling between convection and conduction into consideration [2–4].

In this paper natural or forced convection heat transfer from a straight fin of variable cross section is considered (Fig. 1). A simple procedure is presented, where the coupling between surface temperature and convection has been treated approximately in the case of forced and natural convection. Finally, general efficiency charts of straight fins with different kinds of cross sections are presented.

2. FORMULATION OF THE PROBLEM AND GOVERNING EQUATIONS

In the present investigation, which is an extension of the author's previous paper [4], an analysis is made of the convection heat transfer of a vertical fin of variable thickness. A schematic diagram of the situation under study is shown in Fig. 1 where a straight fin of transverse length l and variable thickness $2t(y)$ is presented. The base temperature is uniform and equal to T_w and the fin is cooled by a fluid at temperature T_∞.

The fin can be cooled either by natural or forced convection. In the case of natural convection only vertical fins are considered. For forced convection the free-stream velocity is equal to U_∞ and both laminar and turbulent boundary layers are included in the analysis.

The energy balance of the fin in Fig. 1 can be expressed as

$$k\frac{\partial}{\partial y}\left(t\frac{\partial T}{\partial y}\right) - q(x) = 0 \tag{1}$$

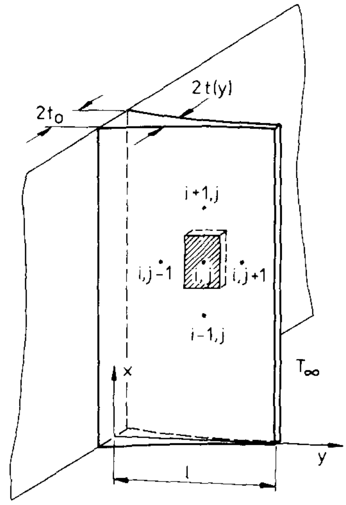

FIG. 1. Schematic diagram of straight fins.

where $q(x)$ is the convective heat flux and depends only on the streamwise temperature distribution. This is the assumption Patankar and Sparrow made in analysing a condensate film [3] and it was used in ref. [4].

Forced convection

For forced convection the connection between the heat flux $q(x)$ and arbitrarily varying surface temperature can be expressed quite accurately for $Pr > 0.6$ [5]

$$q(x) = C\frac{k_f}{x} Re^m\, Pr^n \int_0^x [1-(s/x)^\gamma]^{-\alpha}\, \mathrm{d}T_s(s). \tag{2}$$

The constants in equation (2) are for the laminar boundary layer: $C = 0.332$, $m = 1/2$, $n = 1/3$, $\gamma = 3/4$ and $\alpha = 1/3$. The corresponding values in the case of the turbulent boundary layer are: $C = 0.0296$, $m = 4/5$, $n = 3/5$, $\gamma = 9/10$ and $\alpha = 1/9$.

Natural convection

In the case of natural convection from a vertical surface it is difficult to find the connection between heat flux and surface temperature. Raithby and Hollands have succeeded by using the analogy between a condensate film and the inner part of a laminar boundary layer to relate heat flux and surface temperature for $Pr > 0.6$ [6] as

$$q(x) = ck_f\left[\frac{g\beta}{\nu^2}Pr\right]^{1/4}\Delta T^{5/3}\left[\int_0^x \Delta T^{5/3}\,\mathrm{d}x\right]^{-1/4}. \tag{3}$$

The constant c is obtained very accurately from equation [7]

$$c = \frac{0.503}{[1+(0.492/Pr)^{9/10}]^{4/9}}. \tag{4}$$

For a turbulent boundary layer in natural convection, no equations exist relating heat flux and temperature. It seems, however, that the streamwise variation of surface temperature has an insignificant effect on the heat transfer coefficient. Thus heat transfer of the fin can be obtained by the usual methods.

The iterative method of solving the temperature distribution of a fin governed by equations (1) and (2) or equations (1) and (3) is presented in ref. [4].

3. RESULTS

In practice the most interesting thing, when dealing with applications, is the total heat flux that the fin can transfer. When the temperature distribution is solved the overall rate of heat transfer, ϕ, from the other side of the fin is obtained by integrating over the fin height from 0 to x

$$\phi = \int_0^x kt_0\left(\frac{\partial T}{\partial y}\right)\mathrm{d}x. \tag{5}$$

Very general and valuable results are obtained if the actual heat transfer of the fin is compared with that of an ideal isothermal fin and use is made of nondimensional variables.

Forced convection

When, in the case of forced convection, use is made of nondimensional variables,

$$\theta = \frac{T-T_\infty}{T_w-T_\infty}, \tag{6}$$

$$y_* = y/l, \tag{7}$$

$$X_* = \frac{1}{C}\,\frac{kt_0}{k_f l^2}\,\frac{x}{Re^m Pr^n}, \tag{8}$$

the ratio of the actual heat transfer from equation (5) and the total heat transfer of the ideal isothermal fin

$$\phi_i = C\frac{k_f}{m}Re^m Pr^n \Delta T_w l \tag{9}$$

takes the form

$$\eta = \frac{\phi}{\phi_i} = \frac{m}{1-m}\,X_*^{m/(m-1)}\int_0^{X_*} X_*^{m/(1-m)}\left(\frac{\partial\theta}{\partial y_*}\right)_0 \mathrm{d}X_*. \tag{10}$$

The efficiency charts of different cross-sections are plotted in Figs. 2 and 3, when the fin half thickness is determined by

$$t = t_0[1-(y/l)]^p \tag{11}$$

and p is equal to 0, 1/2, 1, 3/2 and 2.

A comparison between the previous coupled solutions above and conventional fin theory can be obtained if the heat transfer coefficient is assumed constant and equal to the average heat transfer coefficient $\bar{h}$ of an isothermal fin obtained from equation (9). In that case the total heat transfer of a plate fin [$p = 0$ in equation (11)] is

$$\phi = \left(\frac{\bar{h}}{kt}\right)^{1/2} kxt_0\Delta T_w \tanh\left(\frac{\bar{h}l^2}{kt}\right)^{1/2}. \tag{12}$$

When the non-dimensional variable defined by equation (8) is used, the efficiency of a plate fin in the case of forced convection gives

$$\eta = (mX_*)^{1/2}\tanh\left(\frac{1}{mX_*}\right)^{1/2}. \tag{13}$$

The result (13) is shown in Fig. 4 for a laminar and turbulent boundary layer together with the coupled solution. Also in Fig. 4 are presented conventional efficiency charts of a

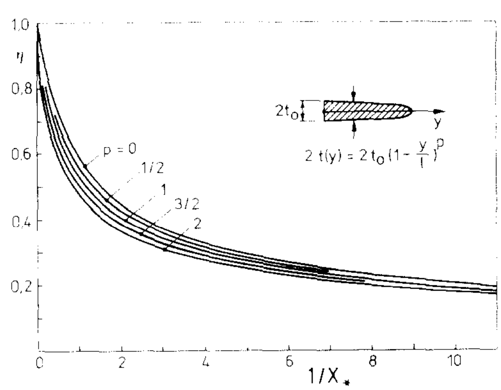

FIG. 2. Performance of straight fins. Forced convection and laminar boundary layer.

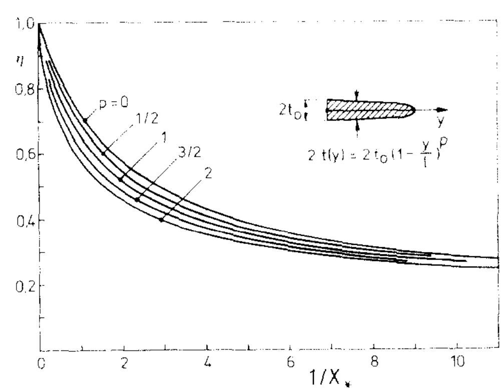

FIG. 3. Performance of straight fins. Forced convection and turbulent boundary layer.

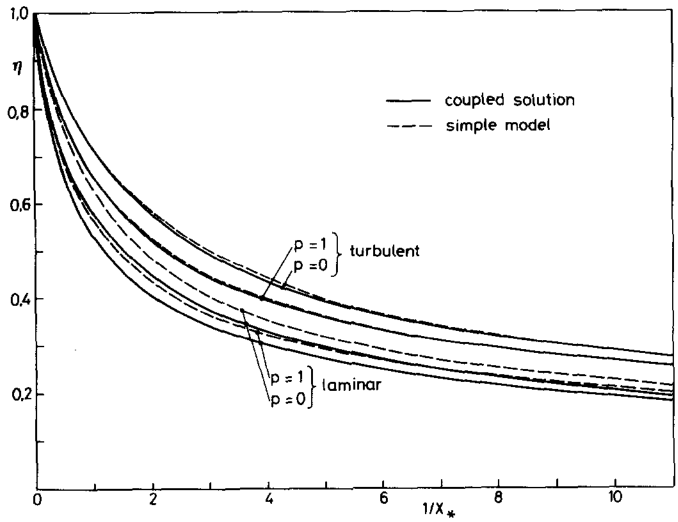

Fig. 4. Comparison between coupled and simplified models in the case of forced convection for plate ($p = 0$) and triangular fins ($p = 1$).

triangular fin ($p = 1$) defined by

$$\eta = (mX_*)^{1/2} \frac{I_1\left(\dfrac{4}{mX_*}\right)^{1/2}}{I_0\left(\dfrac{4}{mX_*}\right)^{1/2}} \tag{14}$$

where I_1 and I_0 are Bessel functions.

It is seen from Fig. 4 that for a laminar boundary layer there is a clear difference between simplified and accurate results. For a turbulent boundary layer results are almost equal. In any case simple theory overestimates total heat transfer.

Natural convection

In the case of natural convection fin efficiency is obtained when the actual total heat transfer is compared with the overall heat transfer of an ideal fin

$$\phi_i = \frac{4}{3} c k_f \left(\frac{g\beta}{\nu^2} Pr\right)^{1/4} \Delta T_w^{5/4} x^{3/4} l. \tag{15}$$

Using the non-dimensional variable

$$x_* = \left(\frac{1}{c^4}\right)\left(\frac{\nu^2}{g\beta Pr}\right)\frac{1}{\Delta T_w}\left(\frac{k t_0}{k_f l^2}\right)^4 x, \tag{16}$$

together with the variables of equation (6) and (7), the fin efficiency of natural convection can be expressed as

$$\eta = \frac{3}{4} x_*^{-3/4} \int_0^{x_*} \left(\frac{\partial \theta}{\partial y_*}\right) \mathrm{d}x_*. \tag{17}$$

The results of equation (17) are shown in Fig. 5 for different fin shapes.

Also for natural convection a comparison can be made with a simple theory. The easiest assumption is that $\bar{h}$ is equal to the average heat transfer coefficient corresponding to the base-to-ambient temperature difference $T_w - T_\infty$. It follows then that the efficiency of a plate fin is using the variable (16)

$$\eta = \left(\frac{3}{4} x_*^{1/4}\right)^{1/2} \tanh\left(\frac{4}{3} \frac{1}{x_*^{1/4}}\right)^{1/2}. \tag{18}$$

For a triangular fin the corresponding result is

$$\eta = \left(\frac{3}{4} x_*^{1/4}\right)^{1/2} \frac{I_1\left(\dfrac{16}{3} \dfrac{1}{x_*^{1/4}}\right)^{1/2}}{I_0\left(\dfrac{16}{3} \dfrac{1}{x_*^{1/4}}\right)^{1/2}}. \tag{19}$$

The results (18) and (19) are shown in Fig. 6. According to it the simplified theory overestimates heat transfer and the difference between coupled and simple theory is essential.

4. CONCLUSION

This paper presents a heat transfer analysis of straight fins of different shapes. It has been found that the solution of a difficult coupled convection–conduction heat transfer problem can be presented without parameters when use is made of the concept of fin efficiency and non-dimensional variables. Using plotted results, the overall heat transfer of any fin of known geometry can be obtained when the fin is cooled by natural or forced convection.

By comparing coupled solutions with those of conventional fin theory obtained using the average heat transfer coefficients of an isothermal surface, it was found that for a laminar boundary layer there was a noticeable difference. For a turbulent boundary layer the difference was small. In any case, conventional fin theory overestimates total heat transfer.

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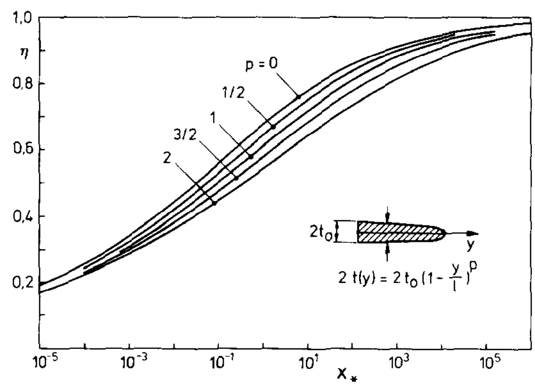

Fig. 5. Performance of straight vertical fins for natural convection.

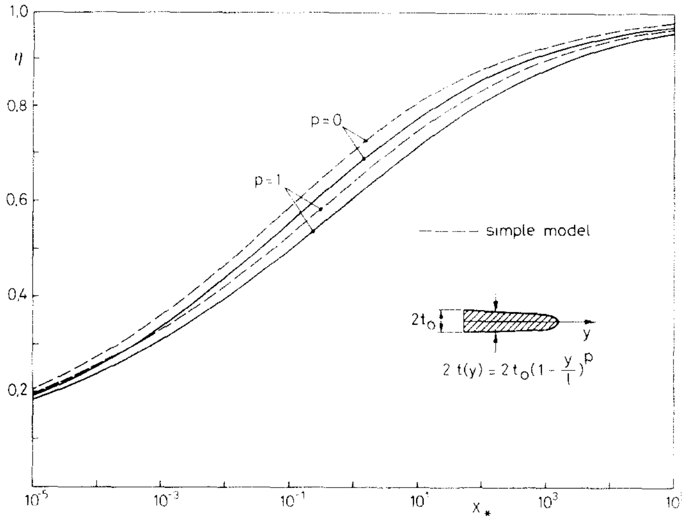

Fig. 6. Comparison between coupled and simple solutions for plate ($p = 0$) and triangular ($p = 1$) fins. Natural convection.

REFERENCES

1. D. Q. Kern and A. O. Krause, *Extended Surface Heat Transfer*. McGraw-Hill, New York (1972).
2. E. M. Sparrow and S. Acharya, A natural convection fin with solution-determined nonmonotonically varying heat transfer coefficient, *J. Heat Transfer* **103**, 218–225 (1981).
3. S. V. Patankar and E. M. Sparrow, Condensation on an extended surface, *J. Heat Transfer* **101**, 434–440 (1979).
4. R. Karvinen, Natural and forced convection heat transfer from a plate fin, *Int. J. Heat Mass Transfer* **24**, 881–885 (1981).
5. W. M. Rohsenow and J. P. Hartnett, *Handbook of Heat Transfer*. McGraw-Hill, New York (1973).
6. G. D. Raithby and K. G. T. Hollands, A general method of obtaining approximate solutions to laminar and turbulent free convection problems, in *Advances in Heat Transfer*, Vol. 11. Academic Press, New York (1975).
7. S. W. Churchill and H. Ozoe, A correlation for laminar free convection from a vertical plate, *J. Heat Transfer* **95**, 540–541 (1973).